Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(6): 13278–13295.
DOI: 10.3934/math.2025595
Received: 20 January 2025
Revised: 14 May 2025
Accepted: 29 May 2025
Published: 10 June 2025

Research article

Minimum span frequency allocation problem: A vector bin packing approach

Ki-Hun Lee[1], **Hyang-Won Lee**[2], **and Bang Chul Jung**[3,*]

[1] Department of AI Convergence, Kunsan National University, Gunsan 54150, South Korea
[2] Department of Computer Science and Engineering, Konkuk University, Seoul 05029, South Korea
[3] Department of Electrical and Computer Engineering, Ajou University, Suwon 16499, South Korea

* **Correspondence:** Email: bcjung@ajou.ac.kr.

Abstract: We revisit the minimum span frequency allocation problem (MS-FAP) to address the spectrum scarcity issue in wireless communication networks. The MS-FAP seeks to minimize the gap (span) between the highest and lowest frequencies used, thereby reducing the total bandwidth required in the network while ensuring the demand of each associated link. We formulate the MS-FAP with the physical interference model as a vector bin packing (VBP) problem on a weighted complete directed graph and then leverage conventional heuristic algorithms based on first-fit decreasing (FFD). Extensive computer simulations and analysis results demonstrate that the FFD-based heuristics outperform the state-of-the-art MS-FA algorithm in both performance and computational complexity. In particular, the FFDsum, an item-centric FFD algorithm, generally achieves the best performance for the MS-FAP. This work is noteworthy in that it is the first to apply VBP to the MS-FAP.

Keywords: first-fit-decreasing; heuristics; minimum span frequency allocation; radio resource management; vector bin packing; wireless network
Mathematics Subject Classification: 90C05, 90C27

1. Introduction

Spectrum scarcity arises because radio frequency is a finite resource that must be shared among numerous communication systems and services, such as mobile networks, wireless local area networks (LANs), satellite communications, radar, and radio/television broadcasting. This issue has been further exacerbated in future wireless networks due to the exponential growth of mobile data traffic, driven by the proliferation of wireless broadband subscriptions and data-intensive applications [1–3]. To address this challenge, several technologies have been investigated. For example, in-band full-duplex (IBFD) communication enables a transceiver to simultaneously transmit and receive signals on the same frequency band, thereby doubling spectral efficiency [2,4]. Dynamic spectrum access (DSA) and cognitive radio (CR) techniques facilitate intelligent spectrum sharing by allowing secondary users to utilize underutilized spectrum without causing harmful interference to primary users [3,5]. Moreover, millimeter-wave (mmWave) and terahertz (THz) bands open new spectrum opportunities, providing abundant bandwidth for immersive communications [6].

From an efficient radio resource management perspective, the traditional minimum-span frequency allocation problem (MS-FAP), also called the minimum span channel assignment problem (MS-CAP), has recently gained renewed attention in the literature [1, 7, 8] as a potential solution to the spectrum scarcity issue in wireless networks. The MS-FAP aims to alleviate the total bandwidth required in the network by minimizing the gap (*span*) between the highest and lowest assigned frequencies while ensuring the quality-of-service (QoS) requirements of each associated link. In particular, it serves as a critical tool for estimating and reporting overall spectrum demand in specific networks or spectrum-leasing scenarios.

1.1. Related work

The MS-FAP is a class of frequency resource allocation strategies in wireless communication systems [9]. Although numerous studies have addressed the frequency allocation problem, only a few have focused specifically on MS-FAP [1, 7, 8, 10–15]. In [10], the authors proposed an iterative algorithm for the MS-FAP in cellular networks, incorporating flexible frequency separation and convex maximization to allocate channels while satisfying co-channel, adjacent-channel, and co-site interference constraints. In [11], the authors formulated the MS-FAP in the multidemand case as an integer linear programming (ILP) problem, and solved it via walk span minimization over a weighted complete directed graph. In [12], two meta-heuristic algorithms were proposed for generating tight lower bounds for the MS-FAP, which eliminate the need for manual clique detection by automatically identifying critical subgraphs through cost functions based on relaxed mathematical programming formulations. In [13], an approximate nondeterministic tree search (ANTS) algorithm was proposed for solving the MS-FAP under multiple interference, combining ant colony optimization (ACO) with local search to iteratively reduce interference within a given spectrum span. In [14], the authors proposed integer and constraint programming approaches for the bandwidth multicoloring problem (BMCP), a generalization of MS-FAP. More recently, in [7], the authors introduced two heuristic algorithms for MS-FAP based on the frequency allocation strategy with node degree reordering (F/DR).

It is noteworthy that all prior works relied on the protocol interference model, where interference is typically modeled through spatial proximity between nodes or frequency separation constraints, rather than accounting for cumulative interference derived from practical received signal strength (RSS), as in the physical interference model. As noted in [8], frequency allocation results derived from the protocol interference model cannot guarantee that each link's practical target QoS constraints are satisfied. In our previous work [1], the optimal MS-FAP with the physical interference model was first formulated as a binary integer linear programming (BILP) problem on a weighted complete directed graph, and a low-complexity minimum span frequency allocation (MS-FA) scheme based on a greedy algorithm was proposed. We have demonstrated that this low-complexity algorithm achieves near-optimal performance in directional antenna-enabled wireless networks. The proposed MS-FA technique has also been extended to an IBFD-enabled backhaul network in the literature [15].

Furthermore, a *distributed* MS-FA technique based on multi-agent reinforcement learning (MARL) was designed in [8] for unmanned aerial vehicle (UAV)-enabled *dynamic* wireless networks.

1.2. Contributions

We have proved in [1] that the MS-FAP with the physical interference model is NP-hard through the following syllogism based on the existing proposition:

i) The MS-FAP with the physical interference model can be formulated as a vector bin packing (VBP) problem;

ii) VBP problems are known to be NP-hard, even in the one-dimensional case [16];

iii) This implies that the MS-FAP is also NP-hard.

This establishes that the MS-FAP can be naturally aligned with the VBP framework. However, despite the availability of well-studied heuristics for the VBP problem, our previous work has not addressed these conventional algorithms.

In this paper, we provide a more intuitive explanation of how the MS-FAP is formulated as a VBP problem. Subsequently, we apply existing heuristic algorithms designed for the VBP problem to the MS-FAP and examine their performance by comparing them with the state-of-the-art low-complexity algorithm named minFAST proposed in [1]. In particular, our focus is on heuristics [16, 17] rather than approximation algorithms, such as the asymptotic polynomial-time approximation scheme (APTAS) [18, 19], because the latter still has poor scalability for large-scale scenarios with many items or high dimensions.

2. System model

A wireless communication network can be abstracted as a complete directed graph, where each vertex represents a communication node-such as a base station (BS), access point (AP), user equipment (UE), or station (STA)-and each directed edge corresponds to a wireless link between a transmitter-receiver pair. More specifically, a link is defined as a unidirectional data path from a transmitting node to a receiving node. If the receiver is the associated node, the link is referred to as a desired link; otherwise, it constitutes an interference link. In the wireless network, each desired link j ($\in \mathcal{L} = \{1, 2, \ldots, L\}$) is assigned a dedicated frequency (or channel) k ($\in \mathcal{K} = \{1, 2, \ldots, K\}$) with a channel bandwidth of W_j Hz to convey information-bearing signals.

Let $r_{j,i}$ denote the average received signal strength (RSS) at the receiver of link j from the transmitter of link i. This quantity accounts for both desired signal power (when $j = i$) and interference (when $j \neq i$), and depends on factors such as transmission power, antenna gain, and path loss. The modeling of the average RSS is detailed in Section 5.1.2. We adopt the physical interference model, in which interference between links is captured through an RSS value rather than a binary indicator of interference presence, as in the protocol interference model. A receiver experiences aggregate interference from all other transmitters operating on the same frequency, which is quantified by the sum of their RSS values. This provides a foundation for expressing system-level signal-to-interference-plus-noise ratio (SINR) constraints that account for realistic inter-link interference.

3. Problem statement

In [1], a *centralized* minimum span frequency allocation problem (MS-FAP) considering the physical interference model has been formulated as a binary integer linear programming (BILP) problem on a weighted directed graph as follows:

$$\underset{z_k,\, z_{j,k}}{\text{minimize}} \quad \sum_{k\in\mathcal{K}} 2^{k-1} z_k \tag{3.1a}$$

$$\text{subject to} \quad z_k \geq \frac{1}{|\mathcal{L}|}\sum_{j\in\mathcal{L}} z_{j,k}, \qquad \forall k\in\mathcal{K}, \tag{3.1b}$$

$$\sum_{k\in\mathcal{K}} z_{j,k} = 1, \qquad \forall j\in\mathcal{L}, \tag{3.1c}$$

$$\alpha_j z_{j,k} + B(1-z_{j,k}) \geq \sum_{i\in\mathcal{L}\setminus j} r_{j,i} z_{i,k}, \qquad \forall j\in\mathcal{L}, \forall k\in\mathcal{K}, \tag{3.1d}$$

where $\mathcal{L}$ (= $\{1,2,\ldots,L\}$) and $\mathcal{K}$ (= $\{1,2,\ldots,K\}$) denote the sets of all associated (desired) links and available frequency (channel) indices in the network, respectively, and $r_{j,i}$ represents the average RSS from the transmitter of link i to the receiver of link j. Note that it is the desired link if $j = i$; otherwise, it is the interference link. In constraint (3.1d), B is a sufficiently large number, e.g., $B \geq \max_j \sum_{i\neq j} r_{j,i}$, and $\alpha_j \triangleq r_{j,j}/\underline{\gamma}_j - N_0 W_j$, where $\underline{\gamma}_j$, N_0, and W_j denote the target SINR, noise spectral density, and channel bandwidth of link j, respectively. Furthermore, z_k and $z_{j,k}$ denote the binary decision variables, each defined as

$$z_k = \begin{cases} 1, & \text{if frequency } k \text{ is used at any link,} \\ 0, & \text{otherwise,} \end{cases} \tag{3.2}$$

and

$$z_{j,k} = \begin{cases} 1, & \text{if frequency } k \text{ is allocated to link } j, \\ 0, & \text{otherwise.} \end{cases} \tag{3.3}$$

The objective function (3.1a) aims to minimize the total bandwidth required in the network by prioritizing the use of lower frequency indices over higher ones. This is because the *span* is defined as the product of the channel bandwidth and the index gap between the highest and lowest assigned frequencies. By assigning frequencies starting from the lowest index, the optimization problem is formulated to minimize total spectrum consumption. Constraint (3.1b) states that frequency k is used if it is allocated to any link in the network. Constraint (3.1c) mandates that each link is assigned exactly one frequency. Finally, Constraint (3.1d) incorporates the QoS requirements for each link j, represented by the SINR threshold derived from Shannon's channel capacity theorem. To be specific, $\underline{\gamma}_j$ can be given by

$$\underline{\gamma}_j = 2^{\overline{R}_j/W_j} - 1, \tag{3.4}$$

by rearranging

$$\overline{R}_j = W_j \log_2\left(1 + \frac{r_{j,j}}{\sum\limits_{i\in\mathcal{L}\setminus j} r_{j,i}\mathbf{1}(f_j = f_i) + N_0 W_j}\right) = W_j \log_2\left(1+\underline{\gamma}_j\right),$$

where $\mathbf{1}(f_j = f_i)$ is an indicator function that equals one if $f_j = f_i$ and zero otherwise. Here, f_j denotes the frequency index allocated to link j. That is, in this paper, we consider only co-channel interference, which arises when two or more links operate on the same frequency. This is a practical assumption, as real-world wireless communication systems typically allocate guard bands at both ends of each channel to suppress adjacent-channel leakage. Meanwhile, the aggregate interference experienced by a receiver increases with the number of transmitters sharing the same frequency, and is quantified by the sum of $r_{j,i}$. Additionally, $\overline{R}_j$ indicates the target transmission rate of link j.

We provide a simple example involving four associated links in a wireless network to illustrate the optimization problem in (3.1a)–(3.1d).

Example 1. *Consider a wireless network with four nodes forming two bidirectional communication pairs:*

- *Link 1: Node 1 → Node 2.*
- *Link 2: Node 2 → Node 1.*
- *Link 3: Node 3 → Node 4.*
- *Link 4: Node 4 → Node 3.*

That is, the set of links is $\mathcal{L} = \{1, 2, 3, 4\}$*. Let* $\mathcal{K} = \{1, 2, 3, 4\}$ *be the set of available frequency indices, assuming a worst-case scenario in which all links use orthogonal channels. For simplicity, we set* $N_0 W_j = 1$*,* $B = 31$*, and* $\underline{\gamma}_j = 1.5$*, for all* $j \in \mathcal{L}$*. The average RSS matrix* $\mathbf{R}$ *is defined as:*

$$\mathbf{R} = \begin{bmatrix} 10 & 20 & 4 & 5 \\ 20 & 10 & 5 & 6 \\ 6 & 5 & 9 & 20 \\ 5 & 4 & 20 & 9 \end{bmatrix},$$

where the diagonal elements represent desired signal powers and the off-diagonal elements are inter-link interference. The value 20 indicates strong self-interference within each communication pair when the nodes transmit and receive simultaneously on the same frequency. First of all, α_j *for all links are computed as* $[\alpha_1, \alpha_2, \alpha_3, \alpha_4] \approx [5.67, 5.67, 5, 5]$*. We now evaluate the following frequency allocation cases:*

- ***Case I (All links share the same frequency)****: When* $z_{j,1} = 1$ *for all* j*, the objective function* (3.1a) *evaluates to* $2^0 = 1$ *since* $z_1 = 1$ *from Constraint* (3.1b)*. Each link is assigned only one frequency index, and all other assignments are zero, i.e.,* $z_{j,k'} = 0$ *for all* $j \in \mathcal{L}$ *and* $k' \in \{2, 3, 4\}$*, satisfying Constraint* (3.1c)*. Intuitively, this allocation violates the SINR constraints for all links, as their SINR values are approximately* $[0.33, 0.31, 0.28, 0.30]$*, which are below the required threshold of 1.5. Therefore, this configuration is infeasible.*
- ***Case II (All links use orthogonal frequencies)****: When all links use orthogonal frequency indices with each other, e.g.,*

$$\mathbf{Z} = [z_{j,k}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

each link's SINR equals α_j*, satisfying the SINR constraints. The value of the objective function becomes* $2^0 + 2^1 + 2^2 + 2^3 = 15$.

- ***Case III (Feasible minimum span solution)**: When Links 1 and 4 use frequency 1, and the other links (Links 2 and 3) use frequency 2, i.e.,*

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

the SINR values are $[1.67, 1.67, 1.5, 1.5]$*, satisfying all constraints. It is noteworthy that Constraint* (3.1d) *holds for all links and frequency indices due to B as follows:*

$$\begin{bmatrix} 5.67 & 0 & 0 & 0 \\ 0 & 5.67 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 31 & 31 & 31 \\ 31 & 0 & 31 & 31 \\ 31 & 0 & 31 & 31 \\ 0 & 31 & 31 & 31 \end{bmatrix} \geq \begin{bmatrix} 5 & 24 & 0 & 0 \\ 26 & 5 & 0 & 0 \\ 26 & 5 & 0 & 0 \\ 5 & 24 & 0 & 0 \end{bmatrix} \qquad (\textit{elementwise}),$$

where the operation of each element corresponds to Constraint (3.1d)*. Furthermore, the value of the objective function evaluates to* $2^0 + 2^1 = 3$*.*
- ***Case IV (Alternate two-frequency allocation)**: When Links 1 and 4 use frequency 3, and Links 2 and 3 use frequency 1, this configuration is also feasible. The value of the objective function becomes* $2^0 + 2^2 = 5$*.*
- ***Case V (Infeasible two-frequency allocation)**: When*

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

the value of objective function is 3. However, Constraint (3.1d) *is violated for some j and k as*

$$\alpha_2 z_{2,2} + B(1 - z_{2,2}) = 5.67 < \sum_{i \in \mathcal{L} \setminus 2} r_{2,i} z_{i,2} = 6, \qquad \textit{when } j = 2, k = 2,$$
$$\alpha_3 z_{3,1} + B(1 - z_{3,1}) = 5 < \sum_{i \in \mathcal{L} \setminus 3} r_{3,i} z_{i,1} = 6, \qquad \textit{when } j = 3, k = 1.$$

*Among the considered cases, **Case III** yields the optimal solution with the minimum objective value while satisfying all constraints.*

3.1. Formulation as vector bin packing (VBP)

Bin packing is a classical combinatorial optimization problem aiming to *pack* a set of items into as few *bins* as possible while ensuring no bin exceeds its capacity. Vector bin packing (VBP) is a non-geometric generalization of multidimensional bin packing [20]. Specifically, in the d-dimensional VBP problem, given items, each of which is a d-dimensional vector, must be assigned to a minimum number of d-dimensional bins such that the sum of the items packed in each bin does not exceed the capacity in any dimension.

As stated in [1, Remark 1], the formulation (3.1a)–(3.1d) can be restated as an L-dimensional VBP problem as follows:

- bin k = frequency k; item j = link j
- each bin k has L resources (dimensions), denoted by $\beta_{k,1}, \ldots, \beta_{k,L}$
- the capacity of each resource in bin k is B
- resources consumed by item j if it is put into bin k are

$$\beta_{k,1} : r_{1,j},\ \beta_{k,2} : r_{2,j}, \ldots,\ \beta_{k,j} : B - \alpha_j, \ldots,\ \beta_{k,L} : r_{L,j}$$

- the goal is to pack all the items (or links) into the minimum number of bins (or frequencies) while the consumption of each resource $\beta_{k,l}$ in each bin remains below the capacity (or SINR requirements are satisfied).

Also, the matrix representation of the formulation is

$$\begin{bmatrix} B-\alpha_1 & r_{1,2} & \cdots & r_{1,L} \\ r_{2,1} & B-\alpha_2 & \cdots & r_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ r_{L,1} & r_{L,2} & \cdots & B-\alpha_L \end{bmatrix} \begin{bmatrix} z_{1,1} & z_{1,2} & \cdots & z_{1,K} \\ z_{2,1} & z_{2,2} & \cdots & z_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ z_{L,1} & z_{L,2} & \cdots & z_{L,K} \end{bmatrix} \leq \begin{bmatrix} B & B & \cdots & B \\ B & B & \cdots & B \\ \vdots & \vdots & \ddots & \vdots \\ B & B & \cdots & B \end{bmatrix}. \tag{3.5}$$

Let $\mathbf{a}_j = [a_{1,j}, a_{2,j}, \ldots, a_{L,j}]^{\mathrm{T}} = [r_{1,j}, \ldots, B-\alpha_j, \ldots, r_{L,j}]^{\mathrm{T}}$ be the jth item vector which is the jth column of the leftmost matrix.

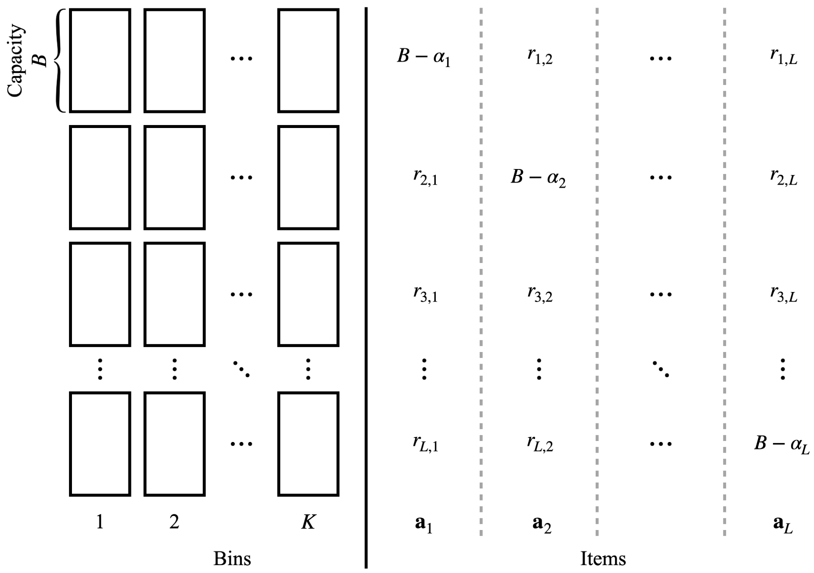


Figure 1. Visualization of the VBP-based MS-FAP.

This VBP problem can be visualized as shown in Figure 1. Suppose that the second item, $\mathbf{a}_2$, is placed into the first bin. The residual capacity of its second dimension then becomes $B-(B-\alpha_2) = \alpha_2$, while the other dimensions retain sufficient residual capacities of $B - r_{j,2}$. If the first item, $\mathbf{a}_1$, is also packed into the bin, the residual capacities of its first and second dimensions are updated to $\alpha_1 - r_{1,2}$ and $\alpha_2 - r_{2,1}$, respectively. More generally, when the jth item is packed into a bin, the jth resource in the bin retains its residual capacity, determined by the RSS of its desired link, the threshold SINR, and the noise power, i.e., $\alpha_j = r_{j,j}/\underline{\gamma}_j - N_0 W_j$. This residual capacity represents the total amount of interference that can be tolerated. On the other hand, if the jth item is not packed into the bin, the jth resource has enough capacity due to the sufficiently large value B ($\geq \max_j \sum_{i \neq j} r_{j,i}$) to accommodate all the items. The formulated VBP problem takes these points into account and aims to minimize the number of bins (frequencies) required to pack all items (links).

4. Heuristics for MS-FAP

As summarized in [16, 17], various algorithms have been developed for VBP problems. Among these, we focus on two heuristics to apply to the MS-FAP: first-fit decreasing (FFD) and bin-centric FFD. The pseudo-codes for these algorithms are outlined in Algorithms 1 and 2, respectively, where $\mathbf{R} = [r_{j,i}] \in \mathbb{R}^{L \times L}$ represents the RSS matrix for all links in the network, K denotes the number of available frequency indices, and $\mathbf{c}(k)$ is the residual (or remaining) capacity vector of the currently open bin k, which will be updated at each iteration.

Algorithm 1 Item-centric FFD

Input: $\mathbf{R} \in \mathbb{R}^{L \times L}$, K, $z_{j,k} = 0, \forall j \in \mathcal{L}, k \in \mathcal{K}$.
Output: $z_{j,k}, \forall j \in \mathcal{L}, k \in \mathcal{K}$.
Initialization: $\mathbf{c}(k) = B\mathbf{1}_{L \times 1}, \forall k \in \mathcal{K}$.
Sort items in decreasing order of size w_j.
for $j = 1$ to L **do**
    for $k = 1$ to K **do**
        if $\mathbf{c}(k) - \mathbf{a}_j \geq \mathbf{0}_{L \times 1}$ **then**
            Place item j into bin k, i.e., update $z_{j,k} = 1$.
            Update $\mathbf{c}(k) \leftarrow \mathbf{c}(k) - \mathbf{a}_j$
            break
        end if
    end for
end for

Algorithm 2 Bin-centric FFD

Input: $\mathbf{R} \in \mathbb{R}^{L \times L}$, K, $z_{j,k} = 0, \forall j \in \mathcal{L}, k \in \mathcal{K}$.
Output: $z_{j,k}, \forall j \in \mathcal{L}, k \in \mathcal{K}$.
Initialization: $\mathcal{I}_{\mathrm{S}} = \mathcal{L}$, $k = 1$, $\mathbf{c}(k) = B\mathbf{1}_{L \times 1}, \forall k \in \mathcal{K}$.
while $\mathcal{I}_{\mathrm{S}} \neq \varnothing$ **do**
    Update $\mathcal{I}_{\mathrm{C}} = \{j | \mathbf{c}(k) - \mathbf{a}_j \geq \mathbf{0}_{L \times 1}, j \in \mathcal{I}_{\mathrm{S}}\}$
    if $\mathcal{I}_{\mathrm{C}} \neq \varnothing$ **then**
        Place largest item j^* into bin k with (4.2), i.e., update $z_{j^*,k} = 1$.
        Update $\mathbf{c}(k) \leftarrow \mathbf{c}(k) - \mathbf{a}_j$
        Update $\mathcal{I}_{\mathrm{S}} \leftarrow \mathcal{I}_{\mathrm{S}} \backslash j^*$
    else
        Update $k \leftarrow k + 1$
    end if
end while

4.1. First-fit decreasing (FFD)

FFD is a well-known heuristic for the one-dimensional bin packing problem. It sorts the given items in decreasing order of size and then assigns them sequentially into bins with sufficient capacity from the first bin, that is, following the first-fit strategy. Before proceeding, consider the following example of FFD applied to a one-dimensional bin packing problem.

Example 2. *Let us consider that there are 14 items, each of size* $\{3, 5, 4, 6, 8, 5, 8, 2, 6, 8, 5, 4, 5, 8\}$, *and 10 bins, each of capacity 12. First of all, the items are sorted in decreasing order as* $\{8, 8, 8, 8, 6, 6, 5, 5, 5, 5, 4, 4, 3, 2\}$. *Afterward, each item is sequentially placed into a bin with sufficient residual capacity from the first bin. Figure 2 shows the result of this procedure, with 14 items packed into seven bins, where the number in parentheses indicates the order in which the items are packed.*

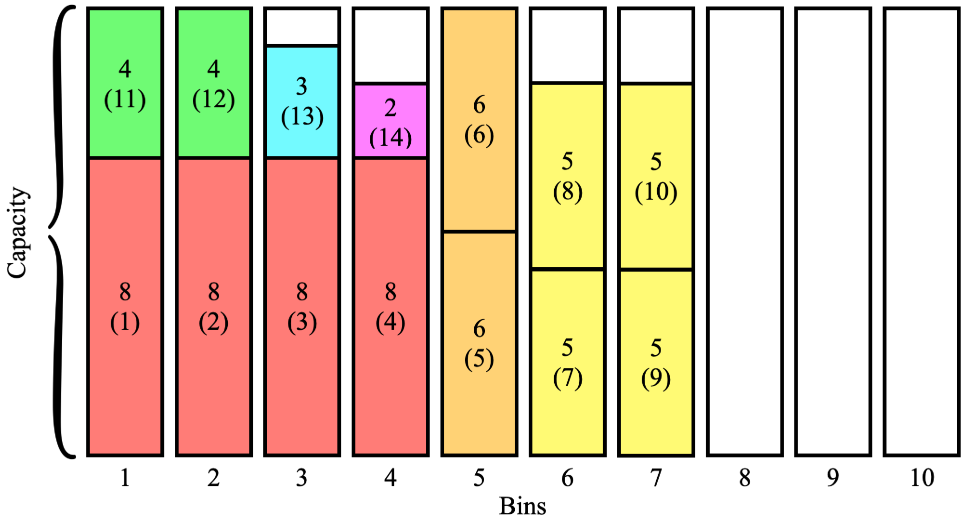


Figure 2. FFD example for one-dimensional bin packing.

Returning to the VBP problem, one might question how to compare the sizes of multidimensional items. To address this, several methods have been proposed for measuring the size of each vector item as follows [21]:

$$w_j = \begin{cases} \max_i a_{i,j}, & \text{FFDmax method,} \\ \sum_{i=1}^{L} a_{i,j}, & \text{FFDsum method,} \\ \prod_{i=1}^{L} a_{i,j}, & \text{FFDprod method.} \end{cases} \tag{4.1}$$

The given item vectors are sorted in descending order according to their sizes, w_j, after which the first-fit algorithm is applied. Note that each item should consider the vector residual capacity of each bin. This approach is referred to as the *item-centric* FFD in [16].

4.2. Bin-centric FFD

This heuristic was proposed to strike bad instances of the aforementioned item-centric FFD methods [16]. It has one open bin at any time and places the *largest* fitting item into the current bin k

at each step. If no more items fit into the bin, a fresh bin is opened, and the process is repeated until all items are packed. For each step, the largest fitting item, denoted by j^*, is determined by a cost function. For example,

$$j^* = \begin{cases} \arg\max_{j \in \mathcal{I}_{\mathsf{C}}} (\mathbf{c}(k) \cdot \mathbf{a}_j), & \text{Dot-product method (DP)}, \\ \arg\min_{j \in \mathcal{I}_{\mathsf{C}}} \|\mathbf{c}(k) - \mathbf{a}_j\|_p, & p\text{-norm method (L}p\text{)}, \end{cases} \tag{4.2}$$

where

$$\mathbf{c}(k) \; (= B\mathbf{1}_{L\times 1} - \sum_{j \in \mathcal{I}_k} \mathbf{a}_j(k))$$

denotes the residual capacity vector of the currently open bin k, and $\mathcal{I}_k$ is a set of items placed in the bin. Furthermore, $\mathcal{I}_{\mathsf{C}}$ ($= \{j | \mathbf{c}(k) - \mathbf{a}_j \geq \mathbf{0}_{L\times 1}, j \in \mathcal{I}_{\mathsf{S}}\}$) represents a set of candidate items that do not violate the bin's capacity constraint in any dimension, and $\mathcal{I}_{\mathsf{S}}$ is a set of items not yet packed. We can observe that its underlying approach is similar to the minFAST algorithm [1].

4.3. Complexity analysis

We analyze and compare the theoretical computational complexities of the heuristics described above using a worst-case analysis approach. First, the computational complexity of the minFAST algorithm has been previously established as $\mathcal{O}(L^4)$ in the literature [1]. Furthermore, the computational complexity of the item-centric FDD, described in Algorithm 1, can be derived as $\mathcal{O}(L^3)$. This is because it involves calculating the residual capacity in lines 5–13 for K L-dimensional bins over L links under the worst-case scenario. Note that the worst case is to exploit orthogonal frequencies for all links, i.e., $K = L$. Similarly, the bin-centric FFD, described in Algorithm 2, has a computational complexity of $\mathcal{O}(L^3)$, as it also performs K L-dimensional vector operations over L links to compute the residual capacity in the while loop. Note that in the worst case, $|\mathcal{I}_{\mathsf{S}}| = L$. We can observe that the FFD-based heuristics have lower computational complexity than the minFAST algorithm.

5. Simulation results

We evaluate the performance of heuristics for VBP when applied to the MS-FAP through extensive computer simulations. We use stochastic network modeling methods to configure various network topologies and compare the heuristics in terms of the average number of frequency indices (bins) required.

5.1. Experimental environment

In this paper, our primary objective is to evaluate and compare heuristic algorithms. To this end, we consider a simple yet practical wireless mesh network (WMN) topology consisting of multiple stationary communicating nodes, as illustrated in Figure 3. It is noteworthy that the presented work is not restricted to such a specific system model. For example, it can be extended to a temporary backbone network formed by multiple hovering unmanned aerial vehicles (UAVs) in areas where traffic demand has surged or natural disasters have occurred. As highlighted in [8], the MS-FAP is particularly suitable for such on-demand networks.

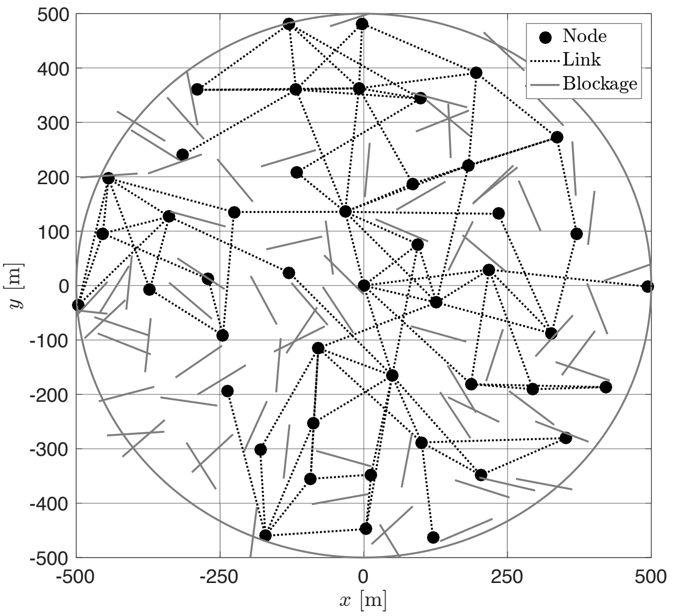


Figure 3. Realization of a wireless mesh network topology.

5.1.1. Spatial model

We consider a two-dimensional circular region with radius R m in which nodes and propagation blockages are randomly distributed. Here, we employ the *germ-grain* model for the obstacles [1, 22]. Specifically, blockages are represented as a sequence of line segments $\Phi_{\mathsf{B}} = \{\mathbf{p}_{\mathsf{B}}, l_{\mathsf{B}}, \theta_{\mathsf{B}}\}$, where $\mathbf{p}_{\mathsf{B}}$, l_{B}, and θ_{B} denote the midpoint location, length, and orientation of each segment, respectively. The midpoints ($\{\mathbf{p}_{\mathsf{B}}\}$) are distributed according to a point Poisson process (PPP) with a density of λ_{B} within the area, and the orientations ($\{\theta_{\mathsf{B}}\}$) are modeled as independent and identically distributed uniform random variables in the range $[0, 2\pi)$.

All nodes are randomly deployed within the same region at a density of λ_{N}, ensuring connectivity even through multi-hop connections. Here, the association policy is based on the signal-to-noise ratio (SNR) threshold. This means that two nodes are connected when the average received SNR is above a certain threshold. We assume that all nodes are equipped with multiple radio transceivers, which allow them to communicate with others simultaneously over different channels. As illustrated in Figure 3, all nodes establish connections with each other, either directly or through multi-hop links. In addition, each link is assigned two frequencies for transmission and reception.

5.1.2. Propagation model

As mentioned in (3.1a)–(3.1d), $r_{j,i}$ denotes the average RSS that the receiver (RX) of link j receives from the transmitter (TX) of link i. It is expressed as

$$r_{j,i} = G_{j,i}^{\mathsf{RX}} P_i^{\mathsf{TX}} G_{j,i}^{\mathsf{TX}} \left(\frac{c}{4\pi f_c}\right)^2 \Phi(\|\mathbf{p}_j - \mathbf{p}_i\|), \ \forall j, i \in \mathcal{L}, \tag{5.1}$$

where $G_{j,i}^{\mathsf{RX}}$ and $G_{j,i}^{\mathsf{TX}}$ denote the antenna gains of the receiver of link j and the transmitter of link i, respectively. Directional antennas are widely employed in WMNs to enhance the RSS of desired links while suppressing interference from other directions [23]. In this paper, we adopt an ideal sectored-pattern array antenna model for the gain pattern defined in [24] as follows:

$$G_{j,i}^{\Psi} = \begin{cases} G_{\mathsf{main}} = \frac{2\pi-(2\pi-\omega)G_{\mathsf{side}}}{\omega}, & \text{if } |\theta_{j,i}^{\Psi}| \leq \frac{\omega}{2}, \\ G_{\mathsf{side}}, & \text{otherwise}, \end{cases} \quad \Psi \in \{\mathsf{TX}, \mathsf{RX}\},$$

where $\omega \in (0, 2\pi]$, G_{main}, and G_{side} denote the beam width, main beam gain, and side lobe gain, respectively, with $0 \leq G_{\mathsf{side}} < 1 < G_{\mathsf{main}}$. Also, $\theta_{j,i}^{\Psi} \in (-\pi, \pi]$ represents the orientation angle between the transmitter of link i and the receiver of link j, relative to the direction of each desired link. When $\Psi = \mathsf{TX}$, the transmit antenna gain considering the angle from the transmitter of link i to the receiver of link j is assigned, while when $\Psi = \mathsf{RX}$, the receive antenna gain considering the angle from the receiver of link j to the transmitter of link i is assigned. The array gains are set to G_{main} for all angles within the main lobe of beam width ω and to G_{side} for the remaining angles, referred to as the side lobe. It is assumed that the beam directions between the transmitter and receiver of each desired link are perfectly aligned ($\theta_{j,j}^{\Psi} = 0,\ \forall j, \Psi$), which determines the beam orientations of the other unintended (interference) links. Furthermore, P_i^{TX} represents the transmit power of link i; c and f_c are the carrier frequency (Hz) and light speed (m/s), respectively. Finally, $\Phi(\|\mathbf{p}_j - \mathbf{p}_i\|)$ denotes the propagation loss as a function of the Euclidean distance between the receiver of link j and the transmitter of link i, where $\mathbf{p}_j$ ($\mathbf{p}_i$) is the two-dimensional Cartesian coordinates of the link j's receiver (the link i's transmitter) node. This is defined as

$$\Phi(\|\mathbf{p}_j - \mathbf{p}_i\|) = \begin{cases} \left(\|\mathbf{p}_j - \mathbf{p}_i\|\right)^{-\alpha_{\mathsf{L}}}, & \text{if } \#(\Phi_{\mathsf{B}} \cap \overline{\mathbf{p}_j, \mathbf{p}_i}) = 0, \\ \left(\|\mathbf{p}_j - \mathbf{p}_i\|\right)^{-\alpha_{\mathsf{N}}}, & \text{if } \#(\Phi_{\mathsf{B}} \cap \overline{\mathbf{p}_j, \mathbf{p}_i}) > 0, \end{cases}$$

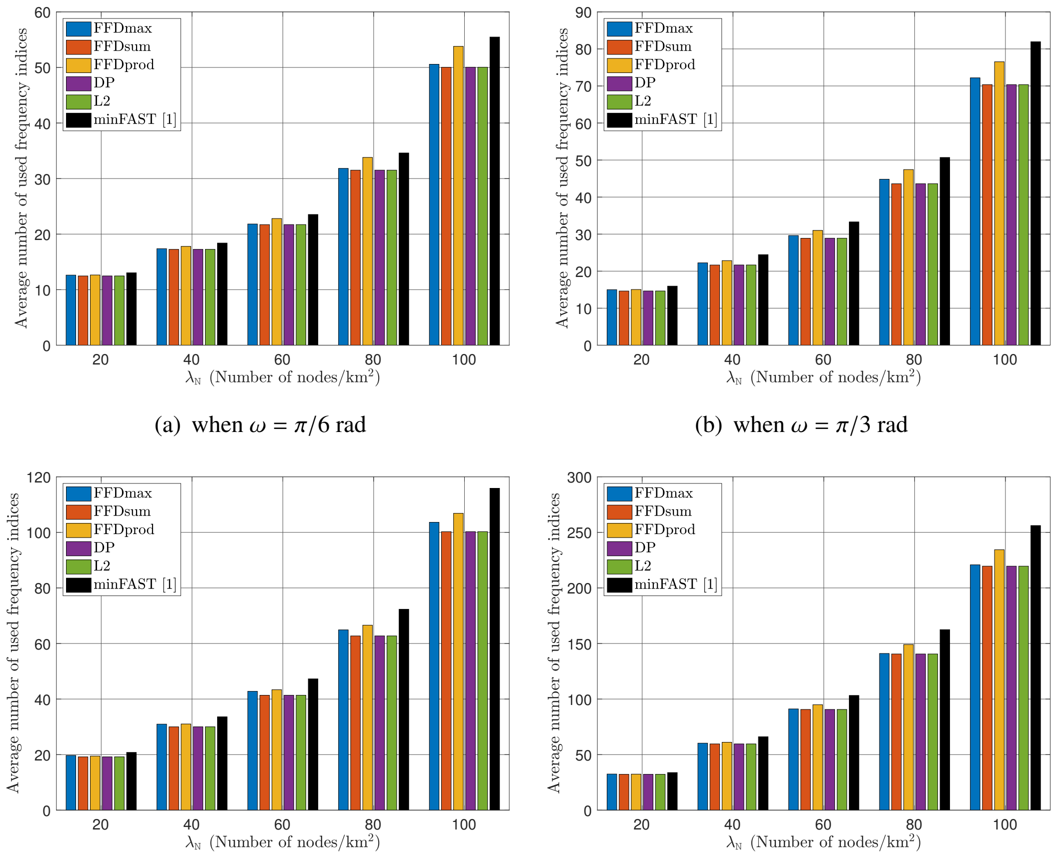
where α_{L} and α_{N} represent the path-loss exponents for line-of-sight (LoS) and non-LoS (NLoS) conditions, respectively, with $\alpha_{\mathsf{L}} < \alpha_{\mathsf{N}}$. The term $\#(\Phi_{\mathsf{B}} \cap \overline{\mathbf{p}_j, \mathbf{p}_i})$ indicates the number of intersections between the elements of Φ_{B} and the line segment $\overline{\mathbf{p}_j, \mathbf{p}_i}$ connecting $\mathbf{p}_j$ and $\mathbf{p}_i$. This means that if there is at least one intersection between the link associating two nodes and a blockage, the link is considered NLoS.

5.2. *Experiment results*

Simulation parameters and values are summarized in Table 1. We set the carrier frequency and each channel bandwidth to 6.525 GHz and 20 MHz, respectively, assuming a spectrum leasing scenario in the 6 GHz unlicensed band [25]. To gradually increase the problem complexity, simulations were conducted with node densities $\lambda_{\mathsf{N}} = \{20, 40, 60, 80, 100\}$. It is worth noting that as the node density increases, the number of nodes and links also grows. Additionally, various beam widths for directional antennas were considered, $\omega \in \{\pi/6, \pi/3, 2\pi/3, 2\pi\}$ rad $= \{30, 60, 120, 360\}^{\circ}$, with a fixed side lobe gain of $G_{\mathsf{side}} = -10$ dBi. Here, $\omega = 2\pi$ corresponds to omnidirectional antennas. The main lobe gain for each beam width is then given by $G_{\mathsf{main}} \approx \{10.37, 7.40, 4.47, 0\}$ dBi, respectively. The SINR threshold, $\underline{\gamma}$, was determined based on the information-theoretic channel capacity (3.4), with a target transmission rate of 200 Mbps for each associated link. Without loss of generality, we assumed identical transmit powers, antenna gain patterns, and SINR thresholds for all links. Note that it is not limited to such specific parameter values. For comparison, we benchmarked a state-of-the-art low-complexity MS-FA algorithm, minFAST, as proposed in [1]. For each node density and beam width, 10,000 distinct WMN topologies were generated, and the heuristics were evaluated in terms of the average number of required frequency indices.

Table 1. Simulation parameters and values.

Parameter	Notation	Value
Radius of area	R	500 m
Node density	λ_{N}	$\{20, 40, 60, 80, 100\}/\text{km}^2$
Blockage parameters	$\lambda_{\mathsf{B}}, l_{\mathsf{B}}$	$100/\text{km}^2$, 100 m
Carrier frequency	f_c	6.525 GHz
Channel bandwidth	W	20 MHz
Path-loss exponents	$\alpha_{\mathsf{L}},\ \alpha_{\mathsf{N}}$	2.0, 3.0
Transmit power of each node	P^{TX}	30 dBm (1 W)
Beam width	ω	$\{\pi/6, \pi/3, 2\pi/3, 2\pi\}$ rad
Side lobe gain	G_{side}	-10 dBi
Main lobe gain	G_{main}	$(2\pi - (2\pi - \omega)G_{\mathsf{side}})/\omega$
Noise power	$N_0 W$	$-$ 174 dBm/Hz+10 $\log_{10} W$+3 dB (noise-figure)
SINR threshold	γ	30 dB



(a) when $\omega = \pi/6$ rad

(b) when $\omega = \pi/3$ rad

(c) when $\omega = 2\pi/3$ rad

(d) when $\omega = 2\pi$ rad (omnidirectional antenna)

Figure 4. The average number of used frequency indices for each heuristic algorithm according to the node density $\lambda_{\mathsf{N}}/\text{km}^2$.

Table 2. The average number of used frequency indices for each heuristic algorithm according to the node density $\lambda_\mathsf{N}/\text{km}^2$ and beam width ω (bold texts indicate the best packing results for the same scenario).

	λ_N	20	40	60	80	100
$\omega = \pi/6$	minFAST [1]	13.038	18.392	23.532	34.605	55.472
	FFDmax	12.622	17.379	21.819	31.829	50.571
	FFDsum	**12.468**	**17.255**	**21.708**	**31.511**	**50.035**
	FFDprod	12.644	17.792	22.805	33.804	53.794
	DP	12.472	17.257	21.717	31.515	50.044
	L2	12.473	17.260	21.717	31.513	50.042
$\omega = \pi/3$	minFAST [1]	15.958	24.461	33.299	50.693	81.950
	FFDmax	15.002	22.243	29.620	44.820	72.200
	FFDsum	**14.633**	**21.656**	**28.882**	**43.599**	70.369
	FFDprod	15.046	22.845	31.000	47.427	76.531
	DP	14.645	21.670	28.898	43.610	**70.367**
	L2	14.645	21.669	28.898	43.609	**70.367**
$\omega = 2\pi/3$	minFAST [1]	20.790	33.616	47.263	72.337	115.876
	FFDmax	19.688	30.973	42.777	64.907	103.623
	FFDsum	**19.191**	**30.004**	**41.365**	**62.719**	**100.260**
	FFDprod	19.470	31.022	43.367	66.592	106.857
	DP	19.198	30.015	41.377	62.728	100.270
	L2	19.197	30.015	41.377	62.727	100.270
$\omega = 2\pi$ (omnidirectional antenna)	minFAST [1]	33.848	66.111	103.244	162.475	256.166
	FFDmax	32.508	60.306	91.105	141.007	220.802
	FFDsum	**32.267**	**59.615**	90.643	140.559	219.604
	FFDprod	32.421	61.099	94.906	149.061	234.394
	DP	32.268	59.618	**90.641**	**140.558**	**219.602**
	L2	32.268	59.618	**90.641**	140.559	**219.602**

Figure 4 and Table 2 present the average number of frequency indices used. Due to the NP-hardness of (3.1a)–(3.1d), obtaining optimal solutions was infeasible. As expected, the number of used frequency indices increases with higher node densities and wider beam widths. It is noteworthy that when each transceiver is equipped with an omnidirectional antenna, i.e., $\omega = 2\pi$, handling inter-node interference becomes significantly more challenging. The results reveal that the conventional FFD-based heuristics for VBP outperform the state-of-the-art low-complexity algorithm, minFAST. In particular, for $\lambda_\mathsf{N} = 100$ and $\omega = 2\pi$, the conventional heuristics require approximately 37 fewer frequency indices compared to minFAST. Furthermore, as discussed in 4.3, the FFD-based heuristics exhibit lower computational complexity than the minFAST algorithm. These findings demonstrate that applying FFD-based heuristics to solve the MS-FAP is more efficient than using the minFAST algorithm in terms of both performance and computational complexity, particularly in high-dimensional scenarios. Finally, we conclude that the FFDsum, an item-centric FFD algorithm, generally achieves the best performance for the MS-FAP.

5.3. Practical applications

The proposed algorithm is designed for (quasi-)static wireless networks in which traffic demands and channel conditions–particularly average RSS values–remain relatively stable over short time intervals. Such environments include wireless backhaul systems, hovering UAV-enabled non-terrestrial networks, and stationary WMNs, where centralized coordination is feasible. In dynamic scenarios, the algorithm can still be applied effectively with appropriate adaptations. For example, dynamic packet transmission characteristics can be captured by adjusting the SINR threshold γ_j to reflect varying data rate requirements. Node mobility can also be accommodated by periodically re-executing the algorithm based on updated network topology and RSS profiles. While instantaneous small-scale channel fading effects cannot be accurately captured within a centralized framework due to signaling overhead and latency constraints, their impact can be mitigated by using average RSS values over short intervals. For highly time-varying channels, distributed algorithms, e.g., [8], may be more appropriate. In summary, the proposed method is well-suited for moderately dynamic and centralized networks. Furthermore, it can serve as a foundation for hybrid resource allocation frameworks that combine centralized planning and distributed fine-tuning.

6. Conclusions

Although the NP-hardness of the MS-FAP has been established through its equivalence to the VBP problem, and well-studied heuristic algorithms for VBP exist, this approach has not been addressed in previous work. In this paper, we reformulated the MS-FAP under the physical interference model as a VBP problem on a weighted complete directed graph and applied conventional FFD-based heuristics, item-centric and bin-centric FFD. Furthermore, we analyzed and compared the computational complexity of the algorithms. The comprehensive experimental and analytical results demonstrated that the FFD-based heuristics outperform the latest MS-FA algorithm in terms of both performance and computational complexity. In particular, the FFDsum, an item-centric FFD algorithm, achieved the best performance for the MS-FAP. These findings suggest that the proposed approach offers a more effective alternative to the state-of-the-art centralized MS-FA algorithm.

Author contributions

Ki-Hun Lee: Writing-original draft, Visualization, Software, Methodology, Investigation, Formal analysis, Data curation; Hyang-Won Lee: Writing-review & editing, Validation, Methodology, Investigation, Formal analysis, Conceptualization; Bang Chul Jung: Writing-review & editing, Validation, Supervision, Resources, Project administration, Funding acquisition, Conceptualization. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This research was supported in part by the Institute of Information & communications Technology Planning & Evaluation (IITP) grant funded by the Korea government (MSIT) (No. RS-2024-00404029, Development of Advanced Duplex System Technologies), in part by the MSIT (Ministry of Science and ICT), Korea, under the ITRC (Information Technology Research Center) support program (IITP-2024-RS-2024-00436406) supervised by the IITP, and in part by the IITP Grant funded by the Korea Government (MSIP, Development of Cube Satellites Based on Core Technologies in Low Earth Orbit Satellite Communications) under Grant RS-2024-00396992.

Conflict of interest

The authors declare no conflict of interest.

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